Building a conference center or triangulating a surface

Executive summary

The Alukonstruct Ltd, located in Szeged, Hungary, is building surfaces from steel support beams and glass triangles. To construct such surfaces is a challenge to the engineers, they must take into account among other things the surface's dead load, the weight of the snow, the wind pressure, and even smaller earthquakes. From the mathematicians point of view this is the problem of triangulating the given surface so that quite a number of conditions are met.

Challenge overview

There is a large literature on triangulating regular surfaces like sphere, cone or cylinder, and there are many computer programs to construct such surfaces. Lately the artists and engineers started to experiment with uncommon surfaces. One can easily triangulate a convex surface: spread points all over the surface, take the convex hull of those points and the triangulation is finished. But such a triangulation is not suitable for building a steel-glass surface, it is a much harder engineering problem. The glass tirangles can break easily if they are to large, or the angles of the triangles are too acute. The steel support beams do not give enough support if the triangles are too blunt angled, or the support beams are too long. If the support beams are shorter, then they do give enough support, but building such a surface is wearisome as the support beams must be soldered together, they must be waterproofed, and in this case there is a large number of glass triangles to handle. So the challenge in this problem is to triangulate a give surface so that the angles and the edges of the triangles are statisfying a strict minimal and maximal condition, and (if possible) also fit in an expected, best minimum-maximum range.

Implementation of the initiative

The Alukonstruct Ltd provided the actual surfaces in triangulated form and the necessary conditions to be met. We implemented several methods for triangulating the surface, but only one of those worked as expected. We were in close contact, early results were estimated by the engineers of the company from the engineering point of view, so that we could take this evaluation into account during the implementation.

The problem

We received the surface in triangulated form, but this triangulation was not appropriate for building the steel-glass surface. The proper triangulations should have the following properties (among others):

- Inside the surface all vertices must be of degree 5 or 6, i.e. only 5 or 6 support beams can meet in a vertex.
- There is a strict and an expected lower and upper limit on the length of the support beams and on the angles of the triangles.
- There are fixed curves and points on the surface (for example: the edge of the surface, the line of a support buttress, the corners on the entrances or the top point of a pier), the triangulation must keep these because they are fundamental parts of the construction.

Naturally, confirming to these restrictions does not mean that the surface can be built in the practice, but surfaces not fulfilling these properties can hardly be constructed.

Results and achievements

The actually working implementation first makes the given (unproper) triangulation finer, and the new triangulation uses vertices from this finer triangulation. The algorithm starts from a point on the surface and adds one triangle at a time. During this procedure one must not cover part of the surface more than once, and eventually all parts of the surface must be covered. After adding a triangle, all vertices of the current triangulation are moved around so that the given lower and upper limits are realized. Also, fixed edges and points are kept, and even holes in the surface can be handled.

Lessons learned and replicability

The program was used for triangulating the Science Building of the University of Pécs, Hungary, although finally they did not construct the building from steel and glass, and the achived triangulation was not used. Lately more complex surfaces came into sight: a building of the Extreme Light Infrastructure of the University of Szeged and the Saransk stadium of the 2018 World Cup.

Contacts, references

References

[1] Alukonstrukt Ltd: http://www.alukonstrukt.hu/

[2] Géza Makay, Bolyai Institute, University of Szeged, Hungary: http://www.math.u-szeged.hu/~makay/

[3] Videos: http://www.math.u-szeged.hu/~makay/avis/

[4] Paper: http://www.model.u-szeged.hu/etc/edoc/imp/

[5] SB, Pécs: http://simonzsolt.wordpress.com/

[6] ELI, Szeged: http://www.u-szeged.hu/sztehirek/

[7] Saransk: http://www.stadiumguide.com/yubileyny/